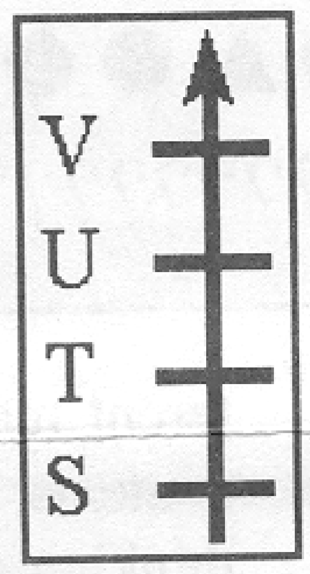
highest to lowest so that they can decide which students get which letter grades. Another reason a ranking may be necessary is that the examination grades may serve as a way to rank students for receiving a departmental prize or scholarship.

It may be helpful at this point to introduce a specific example (Figure 1) to focus our thinking. Here we have students S, T, U and V who have taken three examinations.

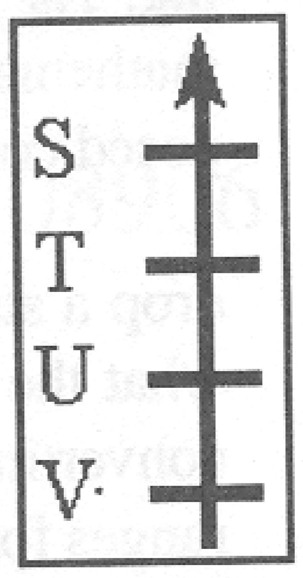
| | S | T | U | V |
|---|---|---|---|---|
| Test 1 | 96 | 55 | 70 | 77 |
| Test 2 | 80 | 90 | 83 | 82 |
| Test 3 | 11 | 84 | 81 | 80 |
| Total (all tests) | 187 | 229 | 234 | 239 |
| Total (lowest grade dropped) | 176 | 174 | 164 | 162 |

Figure 1

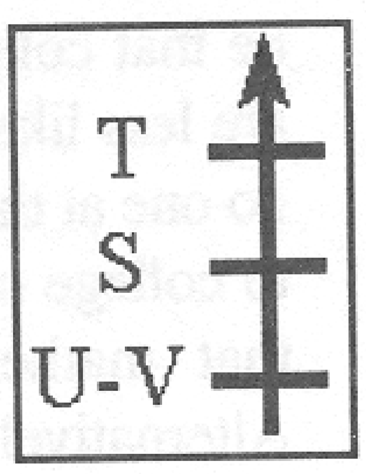
How can we arrive at a ranking of the students? One method of ranking the students is to consider their averages on the three tests. The higher the student's average, the higher rank the student gets. Note that instead of using the highest average, we can instead merely look at the sum of the grades on the three tests. The ranking obtained based on this sum will be exactly the same as the ranking obtained by using the average but will save a fair amount of computation. (This is a nice fact for students to prove.) Based on the data above, the test score sums are: 239, 234, 229, and 187 for V, U, T and S, respectively. This would result in the ranking shown to the right.



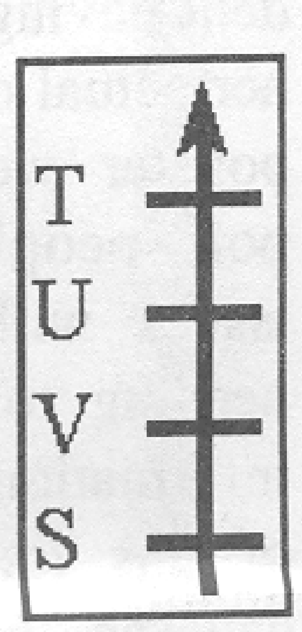

If only the highest ranked student gets the scholarship, the scholarship would go to V. What happens if we drop the lowest grade and rank the students in terms of the sum that they now obtain? The new data is given in Figure 1 and the ranking which results is shown to the right.




Perhaps, unintuitively, not only does V not win the scholarship, but the ranking of the students is now completely reversed! In this ranking S would win the scholarship. (In the context of grading on a curve (with 25% of the students getting A, B, C and D grades), this example shows that dropping the lowest grade might result in a student's getting a D grade instead of an A grade!) These are not the only methods that could be used for giving out the scholarship. For example, we could count the number of exams on which each student got the highest grade and rank the students accordingly. If this is done, S came in highest on one test, T on two, and U and V were never highest on any tests. The ranking which results is shown to the right. Notice how we incorporate the fact that U and V are at the same level. This yields a ranking in which T comes in the highest.




This is reminiscent of situations where individuals rank alternatives and the individual rankings must be combined into a group ranking. On important example is the problem of deciding the unique winner or ranking the candidates who participated in an election. There are a large number of methods (see [1]) that one can use, and the disconcerting fact is that different but seemingly reasonable methods result in different winners (rankings). One popular method for elections is the Borda Count, where alternatives are given credit for how high up on a preference schedule they appear. Thus, 4 points are assigned for a first place, 3 points for a second place, etc. This idea can be adapted to the current context. For each examination, one can see what place the student came in on that examination. Four points are assigned for a first place, 3 points for a second, etc. For example, since S's grades of 96, 80, and 11 were the highest, lowest, and lowest on the examinations, respectively, S would get 4 + 1 + 1 = 6 points. Similarly, T would get 9 points, U would get 8 points, and V would get 7 points. The ranking obtained by this method is shown to the right. How would you change this to allow for dropping the lowest grade?




In this example we have used 4 different methods to produce a ranking and have found 4 different rankings, though the number of different individual winners is only 3. (U is not a winner using any of these methods.) You may wish to have your students construct an example similar to the one produced here where all 4 methods yield different winners. You may also wish to see what other methods your students might develop to rank the students.

We have seen that the issue of whether or not it is fair for an instructor to drop every student's lowest grade raises some interesting questions. When a mathematical algorithm is employed, one expects the algorithm to output a unique answer. Thus, 3x5=15, 2x–3 = 11 has only 7 as its solution, and 1/2 +2/3 = 7/6. Emphasis only on mathematical algorithms gives students the impression that mathematics is a dry, relatively static subject. By adopting a modeling environment, it becomes apparent that mathematics is a more complex subject than students might have otherwise realized. Furthermore, students can see how new mathematics develops and how old mathematics finds new applications.

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[1]. Malkevitch, J. and G. Froelich, *The Mathematical Theory of Elections*, Consortium for Mathematics and Its Applications, Lexington, 1985.