DIMACS
Center for Discrete Mathematics &
Theoretical Computer Science

DIMACS EDUCATIONAL MODULE SERIES

MODULE 08-2
Reed-Solomon Codes: An Application of Linear Algebra
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Module Description Information

- **Title:**

  Reed-Solomon Codes: An Application of Linear Algebra

- **Author(s):**

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- **Abstract:**

This module provides a brief introduction to the theory of error-detecting and error-correcting codes, with special emphasis on the Reed-Solomon codes. Several different methods of defining codes and decoding are presented, and the parameters of these codes and their significance in practice are discussed. Simple, concrete examples are studied first before more general families of codes are considered. Throughout the module, concepts and results from Linear Algebra and (to a lesser degree) Abstract Algebra are used to define and analyze these codes.

- **Informal Description:**

In this module we discuss how one can use linear algebra and finite fields to define and analyze the performance of various error-detecting and error-correcting codes used in Coding Theory, with special attention given to Reed-Solomon codes. These codes are widely used in the production of compact discs (CDs) and digital video disks (DVDs), as well as in cell phone and satellite communications. The module presents examples of Reed-Solomon codes that have smaller dimensions than those used in industry, to simplify the computations and analysis, but which share the same properties as widely used codes. Each section of the module includes exercises for the reader to test his or her understanding and to further explore these codes.

- **Target Audience:**

This module is aimed at the undergraduate students who are studying Linear Algebra or Abstract Algebra, or who are studying Coding Theory as an independent project. Most students registered in these courses are at the sophomore, junior, or senior level.

- **Prerequisites:**

Students are assumed (after some review as provided in the appendix) to be familiar with basic concepts from a linear algebra course such as matrix operations, independent and dependent vectors, span, basis vectors, vector space, dimension, null space, column space, and rank. Finite fields are used, but no previous exposure to finite fields is required.

- **Mathematical Field:**

Linear Algebra, Abstract Algebra.

- **Application Areas:**

Coding Theory

- **Mathematics Subject Classification:**

MSC (2000): 11T71, 94B05, 94B20, 94B35

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- **Other DIMACS modules related to this module:**

None