DIMACS

Center for Discrete Mathematics &
Theoretical Computer Science

DIMACS EDUCATIONAL MODULE SERIES

MODULE 03-4
Planar Linkages: Robot Arms and Carpenters' Rulers

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Nancy Lineken Hagelgans
Department of Mathematics and Computer Science
Ursinus College
Collegeville, PA 19426
email: NHagelgans@Ursinus.edu

DIMACS Center, CoRE Bldg., Rutgers University, 96 Frelinghuysen Road, Piscataway, NJ 08854-8018
TEL: 732-445-5928 • FAX: 732-445-5932 • EMAIL: `center@dimacs.rutgers.edu`
Web: `http://dimacs.rutgers.edu/`

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Module Description Information

- **Title:**

  Planar Linkages: Robot Arms and Carpenters' Rulers

- **Author(s):**

  Nancy Lineken Hagelgans

- **Abstract:**

  A robot arm in the plane is defined as a planar linkage with links connected to form a chain and with one end in a fixed position. We address questions related to the region in the plane that the arm's other end can reach either with or without obstructions to the arm's movement. A carpenters' ruler is another planar linkage in the form of a chain, but a ruler has no fixed end. The question investigated is the minimal folding length of a ruler that has links of different lengths.

  Exercises throughout the module should be completed as they appear in the module. These exercises introduce students to the main ideas through specific examples. More challenging problems appear at the ends of Sections 3, 4, and 5.

- **Target Audience:**

  This module is suitable for mathematically advanced high school students as well as college mathematics students.

- **Prerequisites:**

  The prerequisite knowledge is high school geometry, sigma notation, and proof by mathematical induction.

- **Topics and Goals:**

  The topics are two problems involving planar linkages: the reachability region of a robot arm in the plane and the minimal folding of a carpenters' ruler. The goals are to introduce the concepts of planar linkages and to illustrate proof by mathematical induction in a geometric context.

- **Anticipated Number of Class Meetings:**

  The entire module would take six class meetings. Sections 1 through 3 could be used alone for two class meetings. The first three sections could be followed by either Section 4 or Section 5, each of which would involve two class meetings. On the other hand, the module is suitable for independent student projects with little or no class time devoted to the material.

- **Contact Information:** Nancy Lineken Hagelgans
  Department of Mathematics and Computer Science
  Ursinus College
  Collegeville, PA 19426
  email: NHagelgans@Ursinus.edu

- **Other DIMACS modules related to this module:**

  None at this time