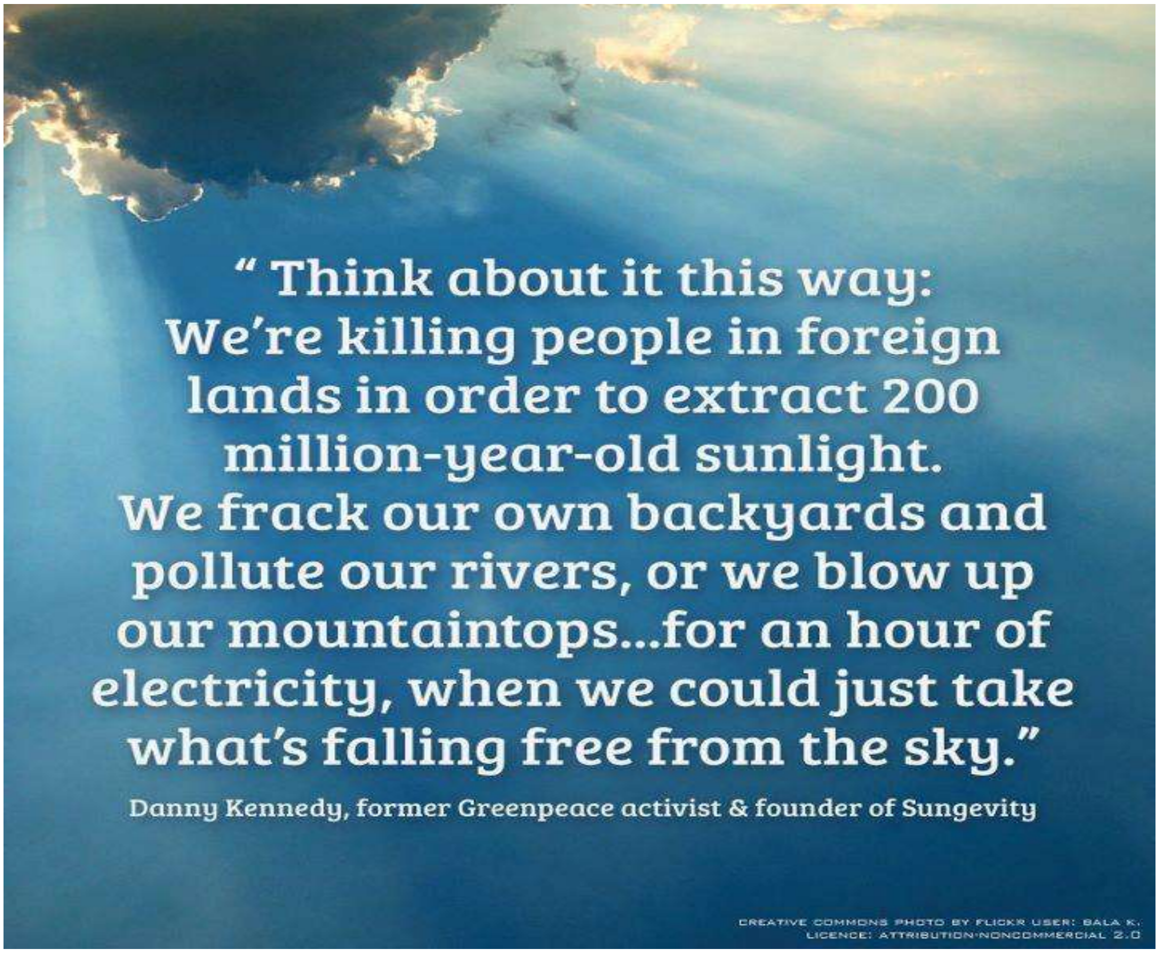
# Passive Solar Module

## A Sustainability Module combining energy lessons and math lessons (trigonometry)



***About this module***

This module is written to be taught in one week of class time, and is separated into five lessons of approximately 45 minutes each. Lessons can be combined to accommodate a block schedule or lengthened to provide class time for homework assignments. Teacher's notes are provided to increase the rigor necessary to suitably challenge students of higher grade and experience levels. The student activities hand-outs are in the appendix.

***In what types of classes could this module be used?***

This module is suitable for students in Environmental Science or Pre-Calculus who have had right triangle trigonometry. Teachers may wish to reduce or eliminate parts of lesson two depending on students' trigonometry experience.

***Prerequisite skills***

- Students should understand basic earth science
- Students should understand basic right triangle trigonometry, that is, apply SOA CAH TOA to basic right triangles.

***Required Materials***

- Big copy paper box (or similar) - 1 per group (of 4)
- 12" x 24" Sheet of cardboard (approximate)
- Insulated cups
- Materials for thermal mass: sand, water, plaster of Paris, wood shavings or other.
- Graph paper
- Flash light or height-adjustable desk lamp
- Heat Lamp or desk lamp with reflective bulb
- Copies of the relevant student worksheets, 1 per student
- Method to watch online animations and videos as a class, such as a computer and projector.

***Summary of Lessons***

Lesson 1: Students learn about passive solar and passive cooling.

Lesson 2: Students review right triangle trigonometry, then apply SOA CAH TOA to basic passive solar problems.

Lesson 3: Students work to understand thermal mass.

Lesson 4: Students learn about declination and how to orient their house due south. They also learn how the angle of sun varies throughout the year.

Lesson 5: Students discover how overhangs are used to block the summer sun.

***Next Generation Science Standards addressed:***

**Content Standard A-Science as Inquiry**
**Abilities Necessary to do Scientific Inquiry**

- Identify questions that can be answered through scientific investigations.
- Design and conduct a scientific investigation.
- Use appropriate tools and techniques to gather, analyze, and interpret data.
- Develop descriptions, explanations, predictions, and models using evidence.
- Think critically and logically to make the relationships between evidence and explanations.
- Recognize and analyze alternative explanations and predictions.
- Communicate scientific procedures and explanations.
- Use mathematics in all aspects of scientific inquiry.

**Understandings about Scientific Inquiry**

- Different kinds of questions suggest different kinds of scientific investigations. Some investigations involve observing and describing objects, organisms, or events; some involve collecting specimens; some involve experiments; some involve seeking more information; some involve discovery of new objects and phenomena; and some involve making models.
- Mathematics is important in all aspects of scientific inquiry
- Technology used to gather data enhances accuracy and allows scientists to analyze and quantify results of investigations.
- Scientific explanations emphasize evidence, have logically consistent arguments, and use scientific principles, models, and theories.
- The scientific community accepts and uses such explanations until displaced by better scientific ones. When such displacement occurs, science advances.

**Content Standard B | *Physical Science***
**Transfer of Energy**

- Energy is a property of many substances and is associated with heat, light, electricity, mechanical motion, sound, nuclei, and the nature of a chemical.
- Energy is transferred in many ways.
- Electrical circuits provide a means of transferring electrical energy when heat, light, sound, and chemical changes are produced.
- The sun is a major source of energy for changes on the earth's surface. The sun loses energy by emitting light. A tiny fraction of that light reaches the earth, transferring energy from the sun to the earth. The sun's energy arrives as light with a range of wavelengths, consisting of visible light, infrared, and ultraviolet radiation. ƒ

**Content Standard D | *Earth and Space Science***
**Energy in the Earth System**

- Earth systems have internal and external sources of energy, both of which create heat. The sun is the major external source of energy. Two primary sources of internal

energy are the decay of radioactive isotopes and the gravitational energy from the earth's original formation.
- Heating of earth's surface and atmosphere by the sun drives convection within the atmosphere and oceans, producing winds and ocean currents.

**Content Standard E | *Science and Technology***

**Abilities of Technological Design**

- Identify a problem or design an opportunity.
- Propose designs and choose between alternative solutions.
- Implement a proposed solution.
- Evaluate the solution and its consequences.
- Communicate the problem, process, and solution.

**Understandings About Science and Technology**

- Creativity, imagination, and a good knowledge base are all required in the work of science and engineering.
- Science and technology are pursued for different purposes. Scientific inquiry is driven by the desire to understand the natural world, and technological design is driven by the need to meet human needs and solve human problems.
- Technology, by its nature, has a more direct effect on society than science because its purpose is to solve human problems, help humans adapt, and fulfill human aspirations.
- Technological solutions may create new problems. Science, by its nature, answers questions that may or may not directly influence humans.
- Sometimes scientific advances challenge people's beliefs and practical explanations concerning various aspects of the world.

***Essential Science Standards addressed:***

HS-ESS3-2. Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.

HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

***Common Core Math Standards addressed:***

G-SRT: Define trigonometric ratios and solve problems involving right triangles

G-MG: Apply geometric concepts in modeling situations

***Note to the instructor:***

Each day, we suggest a "ticket out the door" summarizing activity for the last 3 minutes of class. To do this we suggest using Poll Everywhere. You could use note cards, but if you want to use Poll Everywhere, here is some information to set it up. Poll Everywhere is a site to build questions that students use cell phones to text their answers. Individual teachers should join site and create their own questions. Figure (1) is a screenshot of a multiple-choice question using Poll Everywhere.  For example, if a student chose option ‘a’, then the student would text 100459 to 37607.

We strongly recommend setting up the Poll Everywhere questions before you start the module.

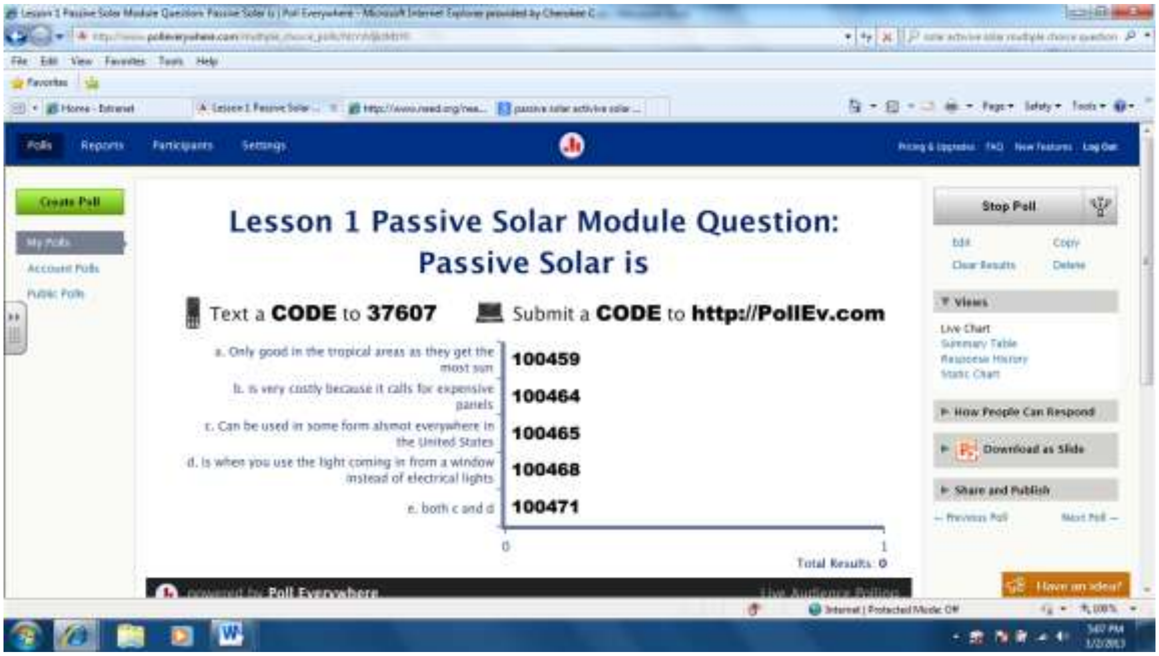



Figure (1):  Screenshot of a multiple-choice question using Poll Everywhere

There are student worksheets for the activities at the end of the module.

**Introduction and Motivation**

**Pre-lesson 1**: Either before coming to class or at the very beginning of the first day, students will watch a portion of the video "Saved by the Sun" which looks at the potential for passive and active solar technologies to be incorporated into residential buildings, and should answer the following questions.
1 – In your own words, describe the advantages that passive solar heating offers the homeowner.
2 – For your state, discuss the possible uses of passive solar technologies and active solar energy generation. Give advantages and disadvantages for the use of passive and active solar in your state.
3 – What do you want to know about passive solar? List questions.

**Lesson 1**: How do we use solar energy?
This lesson is a sequence of activities, beginning with a discussion of passive solar technologies in the context of U.S. Energy Policy. The second activity is to first discuss with a partner about what part of the United States would be appropriate for passive solar. Then individually, students will find a plot of land on which they will do the analysis of the solar potential of an area.

Discussion Question: What are different ways that we use solar energy?

Additional reference material can be found at:
http://www.nrel.gov/
http://www.ases.org/

Active Solar: photovoltaics (production of electricity); solar collectors (production of hot water with possible use for space heating); solar ovens (discussion of use in developing countries where wood is scarce and women spend much time gathering fuel for cooking); industrial use (incineration and other specialized tasks).

Biomass fuels: using plant tissue as a source of energy (ethanol for transportation); wood for home heating and electric generation (Burlington Electric https://www.burlingtonelectric.com/page.php?pid=75&name=mcneil)

Passive solar: using the sun's heat in the winter and rejects the sun's heat in the summer. This is different from active solar heating in that it doesn't involve any mechanical or electrical devices.

Discussion Question: Should we promote the use of solar energy?

Economic Advantages:

1. Introduce the concept of a payback period: in many cases alternative energies are more cost-effective than conventional sources over the long run.
2. Costs are more stable than many conventional sources which are affected by political factors.
3. Many conventional sources of energy cannot be used sustainably (requires discussion of the term) and in some cases are being rapidly depleted.

Environmental Advantages:

1. Solar technologies generate less carbon per unit energy than fossil fuels. Increasing levels of carbon dioxide are associated with combustion of fossil fuels; carbon emissions from solar sources are limited to the production of the equipment if the energy used comes from conventional sources.
2. Transportation of oil and natural gas has historically had significant environmental impacts associated with oil spills and pipeline construction and leakage.
3. Solar technologies generate less waste than nuclear power and has much lower risks of catastrophic problems associated with this technology (discussion of Fukushima, Chernobyl and Three Mile Island).

Political Advantages:

Fossil fuels reserves are concentrated in areas of the world characterized by instability and frequently different value systems than our own.

Challenges to the increased use of solar energy in the US:

Initial investments can be large and payback periods can be long (although this is changing rapidly). The responsibility for the maintenance of the system frequently moves from a centralized provider to individual homes and businesses.

*Activity 1: Is it worth the cost?*

Working with a partner, make a list of the types of energy that you use throughout the day and discuss how these demands could be met by solar power. Next to each of these uses and solutions discuss whether this would be considered an active or a passive form solar energy use.

Payback Time: If a solar collector costs $2000 how long would the payback period be if it produces 60% of a household's hot water if the current cost of hot water is approximately $150 per month?

Assuming the price of hot water produced by oil or natural gas does not change, then the payback time is 23 months (60% of $150 is $90 and 2000/90=23).

In order to do the actual analysis, one must predict future changes in the costs of conventional energy sources; what factors are likely to affect the price of hot water produced by oil or natural gas?

Answer:  Fossil Fuel supplies are limited suggesting that at some point demand will exceed supply creating price instability and ultimately higher costs. The current decline in the price of natural gas associated with increases in the extent of hydraulic fracturing of rock in the northeastern United States stands as an example of possible changes in the cost of conventional fuels associated with technological advances.

Figure 2:  National Renewable Energy Laboratory, Golden, Colorado

*Activity 2:  Location, Location, Location*

In this activity students will discover places in the United States that might be more susceptible to a passive solar building.

In groups of 2, students should answer the following questions:

1. How do the following variables affect a site's suitability for passive solar use?

- Latitude

  Answer: increasing latitude is correlated with increased seasonality and decreased maximum annual mean temperatures. (Note that elevation, ocean currents and atmospheric circulation also affect local climate). The increased seasonality is likely to lead to an increased need for heating, however will also lead to shorter day lengths when the heat is most needed.

- Mean annual high and low temperatures

Answer: Mean annual low temperatures are important as these provide an indication of the need for space heating.

- Average day length in the summer and winter

Answer: As above, long days in summer create the need to reduce the solar gain when the heat is needed less. Short days in the winter afford the minimum time needed to gain the energy from the sun.

For all of the above, the interaction among the variables is critical to determining a site's suitability for passive solar energy and for determining the extent of the return on investment.

2. List at least three other variables that you would use to determine if the additional costs associated with passive solar technologies would affect your decision on whether or not to use these.

3. Visit the United States Geologic Survey's US Map (http://nationalmap.gov/ustopo/index.html) and click on the "Coverage" box.
Rank the following four locations for their likely potential for cost-effective use of passive solar space heating for a home, paying attention to the factors listed your answers to questions 1 and 2 above:

- Key West, Florida
- Nashville, Tennessee
- Golden, Colorado
- Fairbanks, Alaska

There are several possible rankings.

Key West: mean annual low temperature of 64°F suggests that while there is much sun, the need for the heating is low extending the payback period of any investment in solar gain.

All other sites have significant heating needs and thus passive solar technologies are likely to be cost effective. Table 2 shows the values for the key factors noted above, suggesting that Nashville and Golden both have considerable opportunity for effective use of passive solar technologies. Fairbanks would clearly benefit from this, however the shorter day lengths may reduce the effectiveness of these practices in the winter months.

| Location | Mean annual low monthly temperature | Latitude | Day length December 21/June 21 |
|---|---|---|---|
| Nashville | 21 | 36 | 9:42/14:37 |
| Golden | 19 | 39 | 9:21/14:59 |
| Fairbanks | -11 | 65 | 3:42/21:51 |

**Table 1. Comparison of temperature, latitude and day lengths for three cities in the United States**

3 – Using the USGS map, mark on the map one ideal place to build a passive solar home. Using a screen clip tool, paste this map onto this document and justify your choice of location. Note the latitude of your site; we will use this in Lesson 4.

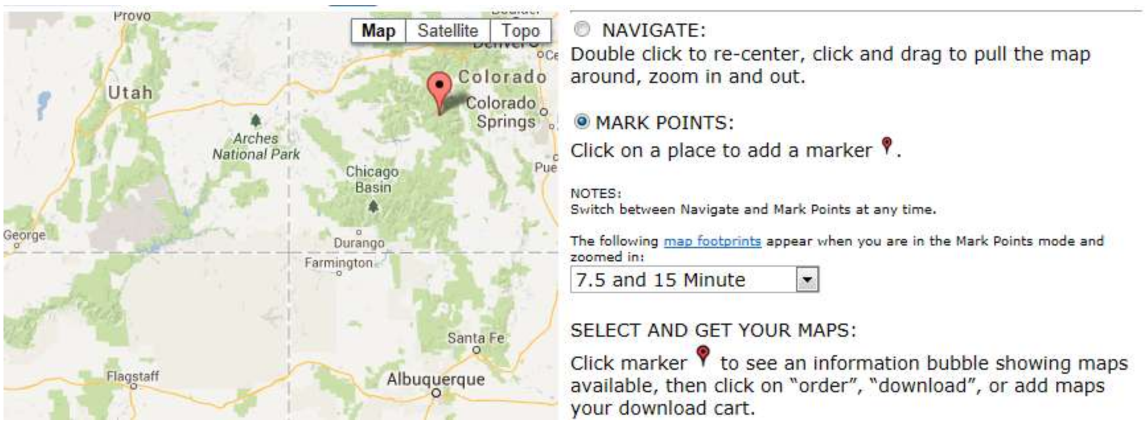


Figure 3. Marking a point on the USGS map

4. Discuss how topography (the shape of the land) affects the suitability of the site for the construction of a passive solar home.

Sites on a southern slope will be suitable for the construction of a home, however a north-facing site may not receive enough sunlight to make the passive solar design feasible.

5. Zoom in on the area selected in question 3 and select a site suitable for home construction (see Figure 4). To do this, click on the "Topo" option in the upper right hand quarter of the map and zoom in until the brown topographic contour lines appear. All USGS maps are oriented with north at the top of the map and show the elevations of points of land using these contour lines. The closer that the lines are to each other the steeper the slope, with water occurring at the bottom of a slope (symbolized by blue lines or polygons) and the high points shown by circles.

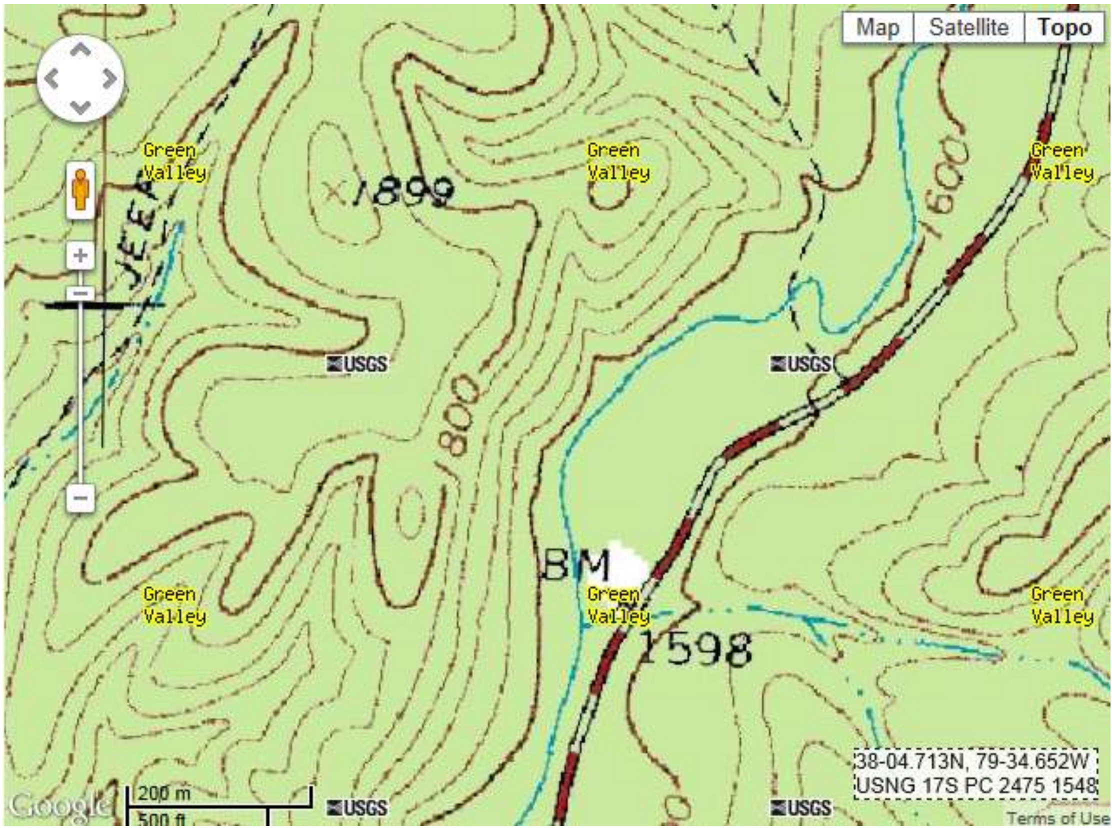


Figure 4. USGS Topographic Map showing contour lines.

Exit Poll Question: Passive solar design is only beneficial in colder climates (True/False).
ANS: False, however sites that require limited heating because of year-round warm temperatures may have longer payback periods.

Homework:
Describe why your house location is appropriate for passive solar design.
(Note that this will be included in their project)

**Lesson 2:** Mathematics Review

This lesson reviews right triangle trigonometry which will be used to calculate the amount of direct sunlight entering a house.

Recall that for a right triangle (Figure 5) the following formulas hold:

$$\sin(\theta) = \frac{opposite}{hypoteneuse} \qquad \cos(\theta) = \frac{adjacent}{hypoteneuse} \qquad \tan(\theta) = \frac{opposite}{adjacent}$$

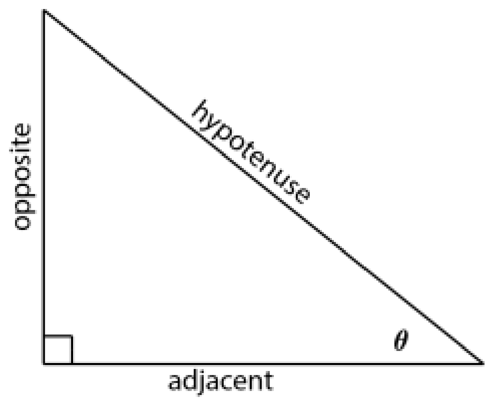



Figure 5: Right triangle with opposite, adjacent and hypotenuse labeled

Also recall that angle of elevation is the angle from the horizontal up to an object while the angle of depression is angle from the horizontal down to an object. Note, however, that they are equal because the two horizontals are parallel lines and the line of sight is the transversal, making the angle of depression and angle of elevation alternate interior angles (which are congruent).

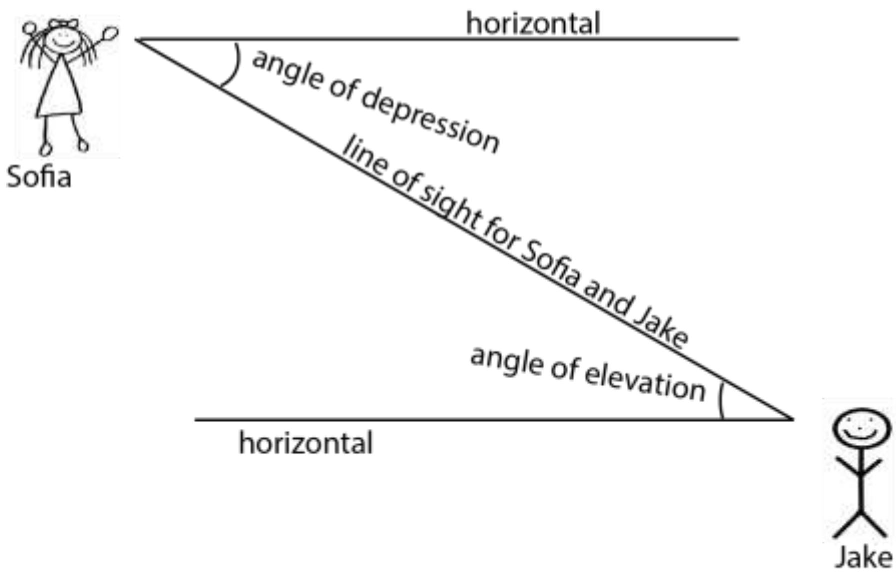
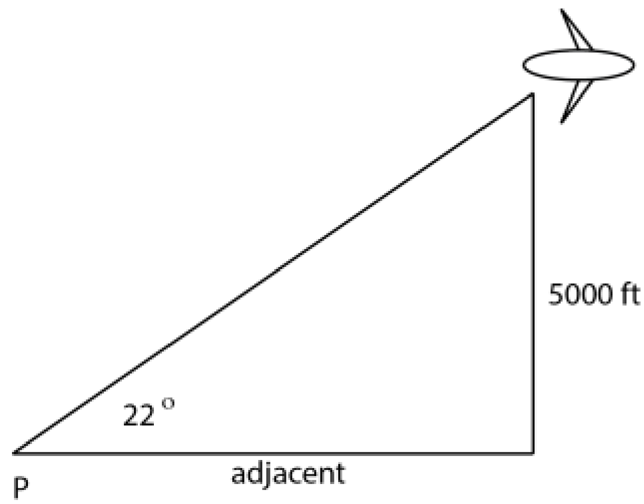



Figure 6: Illustration of angle of depression and angle of elevation

Example 1: The angle of elevation from a point P on the ground to an aircraft flying at an altitude of 5000 feet measures 22°. How far is it from P to the point on the ground directly beneath the aircraft?

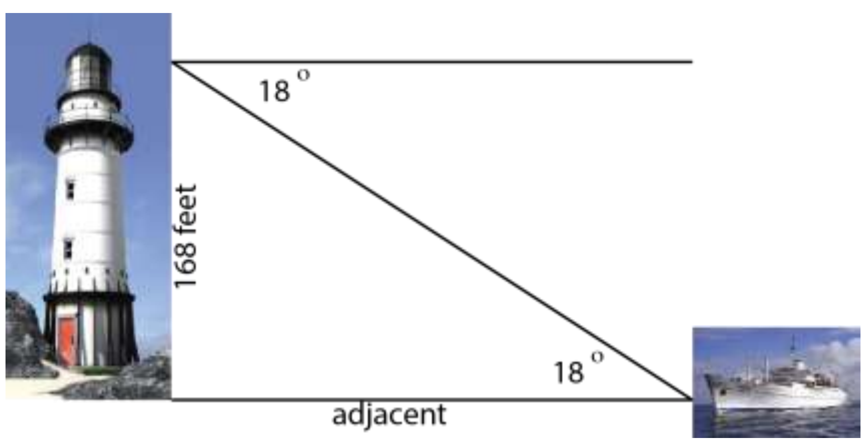
Solution 1: First you should draw a picture, which could look something like:




Then set up an equation. Since you are given the side opposite the angle and want to know the side adjacent, we'll use $\tan(\theta) = \frac{opposite}{adjacent}$. Plugging in the values that we know, we get $\tan(22) = \frac{5000}{adjacent}$. Finally, solving for the adjacent side: $adjacent = \frac{5000}{\tan{(22)}} \approx 12{,}375\ feet$.

Example 2: The angle of depression from the top of a 168-foot lighthouse to a ship sailing is 18°. How far away is the ship from the base of the lighthouse?

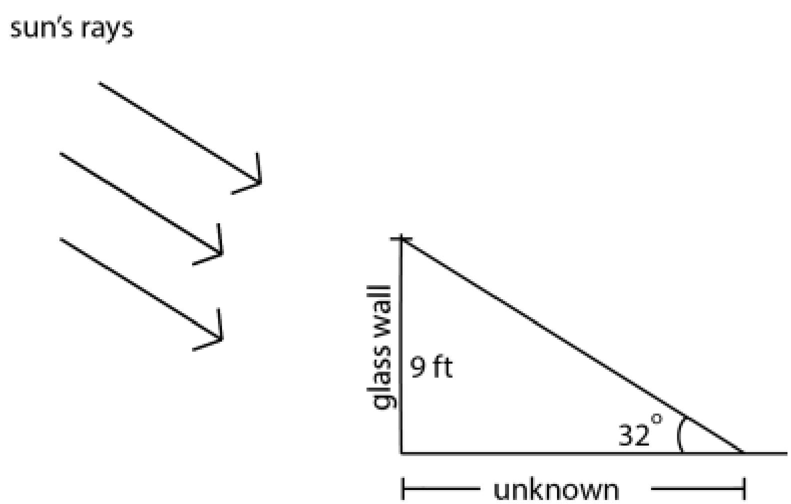
Solution 2:



18° can go in both of those positions because the angle of elevation and depression are equal. Once again, we are solving for the adjacent side and we are given the opposite side: $\tan(18) = \frac{168}{adjacent}$. Thus, $adjacent = \frac{168}{\tan(18)} \approx 517\ feet$.

Example 3: Glass Wall

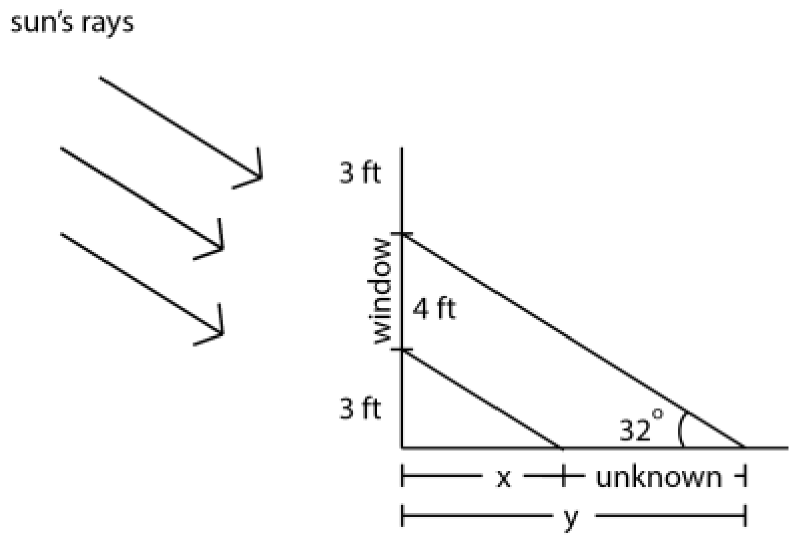
A house has a 9 foot glass wall on the sunny side of the house. How far does the sun come into the room when the sun's rays make an angle of 32° with the floor?




Solution 3: Since the unknown is the adjacent side and we are given the opposite side, we'll use tangent: $\tan(32) = \frac{9}{adjacent}$. Thus, $adjacent = \frac{9}{\tan(32)} \approx 13.6\ feet$. So, if the room was only 12 feet deep, then the direct rays would be hitting the back wall.

Example 4: Passive Solar example

A room in a house is 12 feet deep and the length of the room is 12 feet with a 4-foot tall window that is 3 feet off the ground. If the sun's rays make an angle of 32° with the floor (note that this is the angle of depression and elevation), what floor area is exposed to direct sunlight?




Solution: First, solve for the unknown distance marked in the figure which represents the part of the floor with direct sunlight. To do this we calculate the distance from the wall to the first sun ray (call this distance x) and then the distance between the wall and the second sun ray (call this distance y). To solve for x, we use right triangle trigonometry: $\tan(32) = \frac{3}{x}$, so $x \approx 4.8\ feet$. Similarly, $y \approx 11.2\ feet$. Thus the unknown distance in the figure is 11.2 – 4.8 = 6.4 feet. In order to find the square footage, we would need to know how wide the window is. If the window is 3 feet wide, then the floor area of direct sunlight is (3 feet)(6.4 feet)=19.2 square feet.

There are two worksheets in the appendix. Both worksheets ask the same questions, one has the pictures drawn, the other doesn't. These could be used for homework or in class.

*Activity: Let the Sun Shine In*
This activity will demonstrate the relationship between the sun angle (at noon) at different times of the year with the sunlit area on the floor inside the house.

*Materials needed:* You will need a sheet of cardboard approximately 12" wide and 24" long, a ruler, scissors, marker and graph paper.

*Directions:* Lay out the cardboard as shown in Figures 7 and 8, fold it evenly in thirds, lay it flat and cut a circular hole about ½ inch in diameter in the left pane, 4 inches from the edge. Cut out a rectangle in the right hand pane, leaving about a 1 inch margin around the edge of the gap. Paste a sheet of graph paper in the center section as shown.

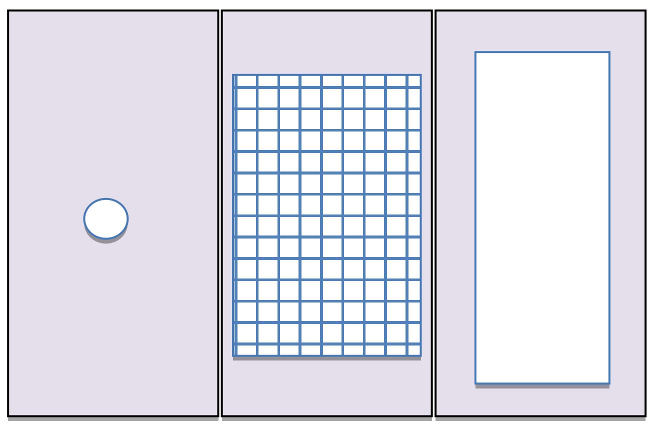


Figure 7. Construction of solar structure

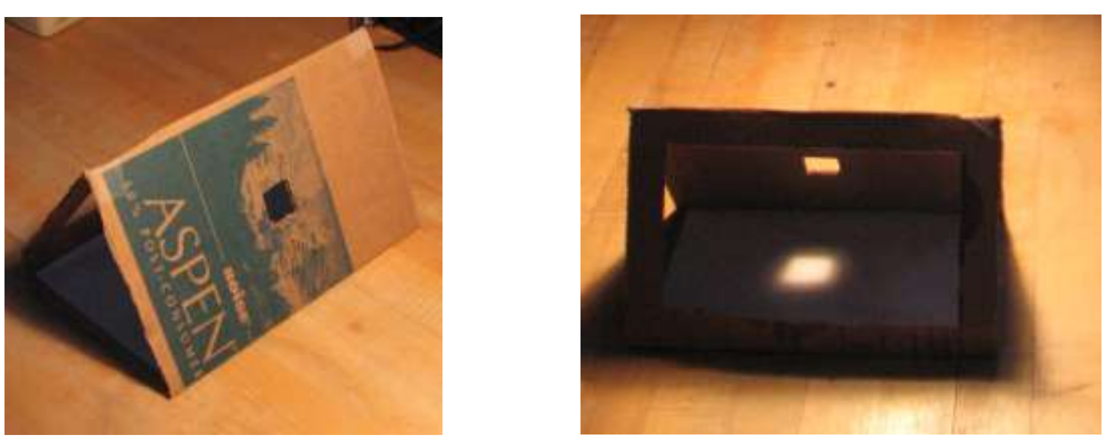



Figure 8: Folded solar structure from 2 different sides

Fold the cardboard into a right triangle and tape together (note that the side with the rectangle cut out will not reach the top), placing it on the table with the graph paper on the floor (see Figure 7 and 8). Attach a flashlight to a ring stand and place it approximately 24" from the structure (Figure 9).

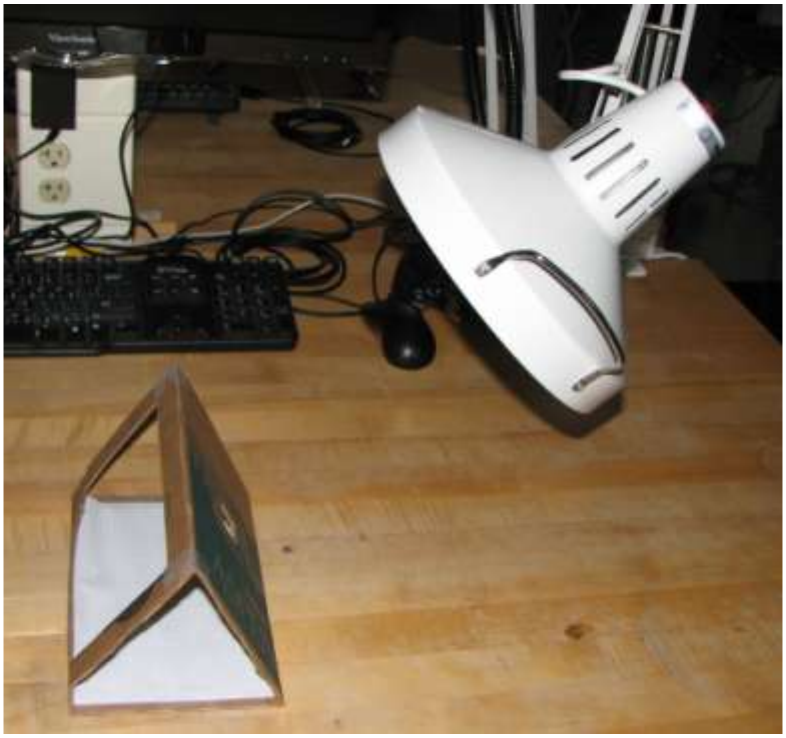


Figure 9: Folded solar structure with light

Questions:
1 - Describe what happens to the sunlit area as the flashlight (sun) moves up.

2 – Do you think the areas would be the same if you moved the window to the bottom of the box? Or the top of the box? Explain.

3 – Quantify the floor area covered by the light shining from the flashlight. To do this, begin with the flashlight at the lowest level of ring stand and move it up until the circle of light produced on the floor touches the far wall. By counting squares on the graph paper, approximate the sun lit area. Record the height and approximate floor lit area in the first row of Table 2. Move the flashlight up the stand until no light shows on the floor and record this height (and 0 for floor lit area) in row 5 of Table 2. Fill in rows 2, 3 and 4 in even increments between the values in rows 1 and 6.

Note: Students could use the area of an ellipse as another way to approximate the floor lit area. The area of an ellipse is π*A*B where A is half the major axis and B is half of the minor axis.

| Height of flashlight above table | Approximate floor area lit |
|---|---|
| | |

| | |
|---|---|
| | |
| | |
| | |
| | |
| | |

Table 2. The effect of the position of the light source on the floor area illuminated

4 – Does this work suggest that there is an optimal angle? In other words, is there an angle that produces the most sunlit area on the floor? If so, what is it?

5 – Compare your answers to mathematics: Using right triangle trigonometry, calculate the angle of the flashlight at each of the heights and determine the expected lit area of the floor. From this, calculate the expected lit floor area. How do your answers compare?

| Calculated Angle | Predicted lit area | Actual Area |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

To calculate the actual area, use the same method demonstrated in example 4 to calculate the length of the sunlit area. Since the "window" is circular, use the area of an ellipse to calculate the sunlit area. For example, consider a 30° angle and a circle with 1/2" diameter cut out 3.5" from the bottom fold. Following example #4, x=3.5/(tan(30))≈6.062 and y= 4/(tan(30))≈6.928. Thus the major axis of the ellipse (sunlit area on the floor) is .866" and the minor axis is 0.5". Then using the formula, π*A*B, we get π*(.866/2)*(.5/2)≈.34 square inches.

6. Both the area and the location of the sunlit area change as the solar angle changes. Use right angle trigonometry to determine the location of this sunlit area at the angles.

| Angle of the sun | Distance of the ellipse from the far wall (in) |
|---|---|
| 15 | |
| 30 | |
| 45 | |
| 60 | |
| 75 | |

7 – Bringing ideas together: Why do we care about how much sunlit area there is? We didn't consider the sunlight that hit the walls. Explain why this might matter.

Exit Poll Questions:

1. The angle of elevation and the angle of depression are equal (True/False). ANS: True
2. The angle of elevation between the window and the sun that let's the most sun in is 70°. ANS: False

Homework:

1 – Worksheet on right triangle trigonometry

2 – Watch this 9 minute Youtube video on thermal mass:
http://www.youtube.com/watch?v=jrVSoOcSSHw. This video illustrates how a source of thermal mass can be used to prevent a home from heating or cooling too much using two model homes.

Questions from video:

1. Which home heats up faster? (ANS: the home without the thermal mass)
2. Which home cools faster? (ANS: the home without the thermal mass)
3. Describe how thermal mass effects temperature fluctuations. (ANS: The thermal mass reduces the temperature fluctuations. In other words, when there is no thermal mass present, the difference between the high temperature and low temperature is bigger than when a thermal mass, such as a tub of water is present. )
4. What was for the thermal mass? (ANS: water)

**Lesson 3**: Thermal mass, or heat capacity is a numerical value that describes an objects ability to store thermal energy. For example, during a hot summer day, the inside of a car can reach over 100 degrees in less than 30 minutes and can reach to over 150 degrees depending on car color, external temperature, etc., but as soon as the sun goes down the temperature inside the car drops. Because the temperature fluctuates a lot, the car has a low thermal mass. In other words, the higher the thermal mass, the better the object is at leveling-out the temperature. In passive solar design, the goal is to use the concept of thermal mass to prevent (or dampen) temperature fluctuations, that is, have a high thermal mass.

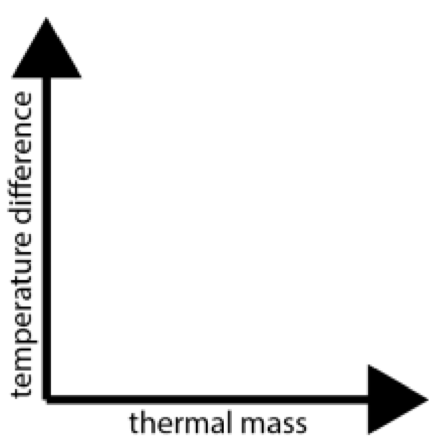
Question: From the video last night, what did they use as their thermal mass? (ANS: water)

From a mathematical point of view, this can be thought of as indirect variation, that is, as the thermal mass increases the amount of temperature fluctuations decreases. For example, if you have concrete floors in your passive solar home, your daily temperature difference (high temperature – low temperature) will be lower than if you had hard wood floors. Examples of thermal mass values are in Table 2.

| Material | Thermal Mass (volumetric heat capacity, KJ/m^3) |
|---|---|
| Water | 4186 |
| Concrete | 2060 |
| Sandstone | 1800 |
| Brick | 1360 |
| Earth Wall (adobe) | 1300 |

Table 3: Examples of thermal mass values (http://www.yourhome.gov.au/technical/fs49.html)

Problem: Sketch a graph that describes the relationship between thermal mass and temperature fluctuations.



*Activity: Three Little Pigs*
This activity investigates the different ways in which standard building materials can affect interior temperatures by comparing several identical interior spaces, represented by small, insulated cups. You will place different types of building materials inside these chests, and observe how quickly they heat up and how the temperatures changes in the cups after the light source is removed.

*Materials needed:* You will need four (or eight if increased insulation is desired), four prepared petri dish tops and four thermometers for each group. Each group will be provided with water, sand, plaster of Paris and wood.

(Teacher preparation required prior to the activity: a hole must be placed in the top of the petri dish which is approximately the same diameter of the thermometers used. This can be done either with a drill or by heating a metal rod or nail and melting a hole in the top of the cover. Wood disks need to be cut using a hole saw kit, selecting a diameter approximately equal to the diameter at the bottom of the cup. If this is impossible, wood shavings can be substituted for the sold wood)

Directions:

1. Fill the cups with 1" of each of the materials provided by your teacher. Cover these with the petri dish covers and place in direct sunlight or beneath a heat lamp (see Figure 10).

2. Record the temperature changes every 15 minutes for one hour.

3. Place the cups in an ice bath and record the change in temperature every 3 minutes until they have cooled to 50 degrees.

4. Plot the data on graph paper for each of the four materials.

Figure 10. Cup with petri dish cover underneath a lamp

Before doing the experiment, students should answer the following questions:

1. Predict which material will cool the air inside the box the most, before the ice chest starts warming up again from the heat gain from outside the box.
2. Will there be a time lag between the boxes? In other words, which box will get to its coldest temperature first?

Once all groups have finished the experiment, they will share their findings with the class. Together, the class should discuss the following questions:

3. Which material heated the cup the fastest?
4. Which material maintained the temperature in the cup the longest?
5. Working with a partner, discuss other considerations that might be important to the selection of a material for thermal mass.

Exit Poll Questions:

1. A high thermal mass means less temperature fluctuations. (True/False) ANS: True
2. Which material (water, brick, aluminum or wood) has the highest thermal mass? ANS: Water

**Lesson 4:** Solar Angles and Introduction to Overhangs

In this lesson, the goals are 1- to understand how the solar angle at different latitudes affects the amount of sun received by a structure and 2 - learn how overhangs can be used to reject sunlight in the summer.

In the United States, the path of the sun is lower in the sky during the winter and higher in the sky during the summer. Figure 11 shows that at noon, the angle of the sun's rays is 30° with the house during the winter and is 76° during the summer.

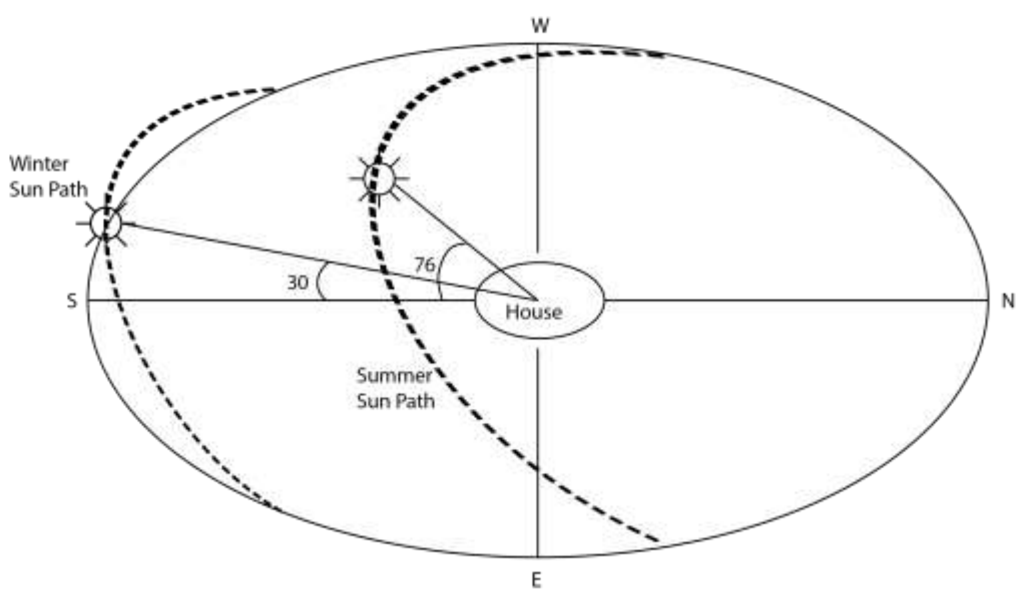



Figure 11: Angles of elevation of the sun's rays at noon during winter (30°) and summer (76°) for Vilas, NC (http://www.susdesign.com/sunangle/)

To help students understand why the path of the sun is lower in the sky during the winter and higher during the summer, they can watch http://www.youtube.com/watch?v=OR8EQ0DWpPw&feature=related or you could explain the content of the video yourself.

*Activity: Calculating Sun Angles*
To find angles for the location that you selected in Lesson 2, use http://www.susdesign.com/sunangle/. Figure 12 is a screenshot of the webpage. First, type in your zip code and click on lookup. This will input your longitude and latitude. Change the date and time then calculate. The altitude angle is the first output. For example, figure 12 demonstrates where the 43° angle came from for the angle during the summer at 9 am.

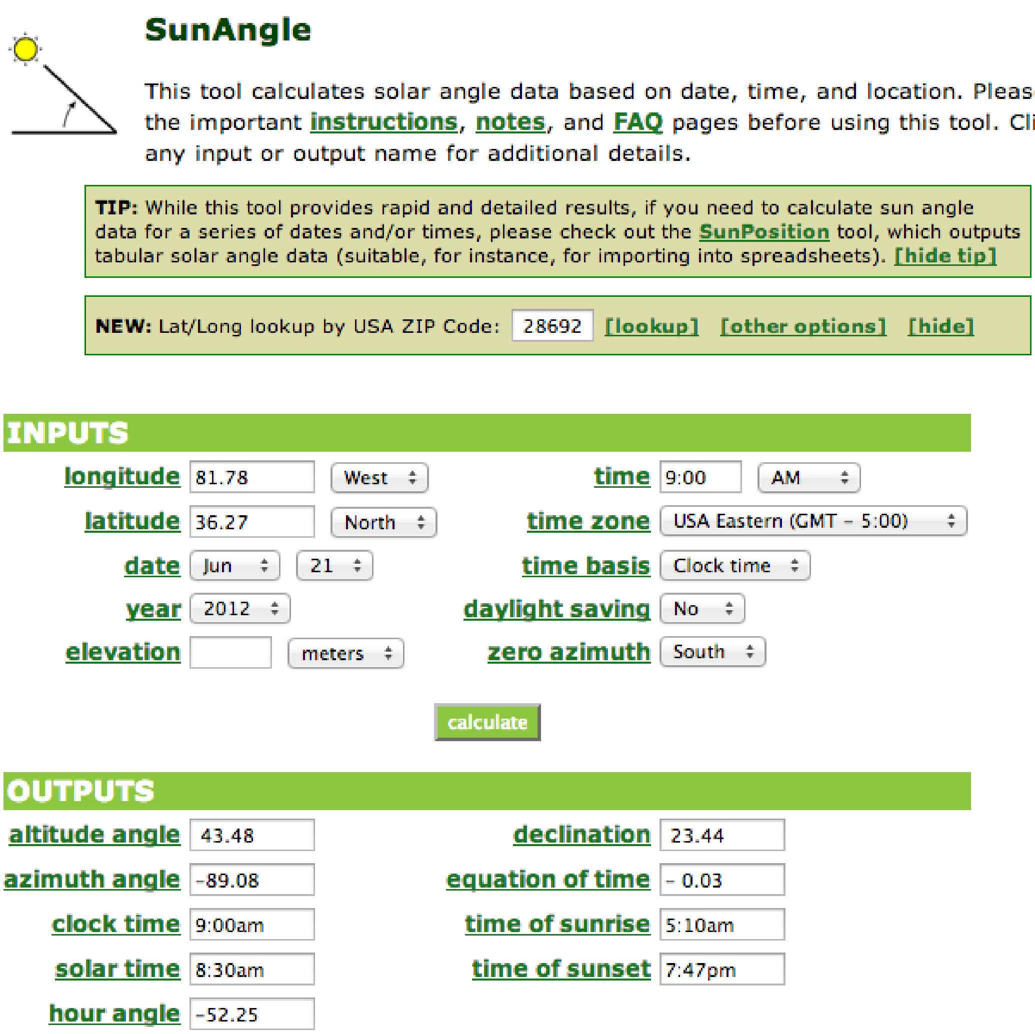



Figure 12: Screen shot from SunAngles (http://www.susdesign.com/sunangle/) demonstrating that the angle of elevation is about 43° at 9am on June 21st in Vilas, NC.

Now that we know how to find the angles, how can we determine how much sun will come into the house in the summer and winter at different times. Table 4 shows the sun angles at 9am, noon and 3pm for both June 21st and December 21st for Vilas, NC. Use the webpage to check a couple of the angles in the table before filling in your own.

| Time | June 21st | December 21st |
|---|---|---|
| 9 am | 43° | 13° |
| Noon | 76° | 30° |
| 3 pm | 55° | 20° |

Table 4: Angles at various times and dates for Vilas, NC

Problem: Fill in Table 5 for the site which you selected for analysis

| Time | June 21st | December 21st |
|---|---|---|
| 9 am | | |
| Noon | | |
| 3 pm | | |

Table 5: Angles at various times and dates for *your location*

Assuming nothing blocks the sun during the summer, how much of your floor would be absorbing light? Ideally, you would like no direct sunlight in your house. What could you use to block out the sun? (Answer could vary – blinds, curtains, overhangs, etc. You may want to include a discussion about the advantages and disadvantages of each.)

Figure 13 shows how an overhang can block the sun's rays. There are several other ways to block the sun, such as shudders or blinds.

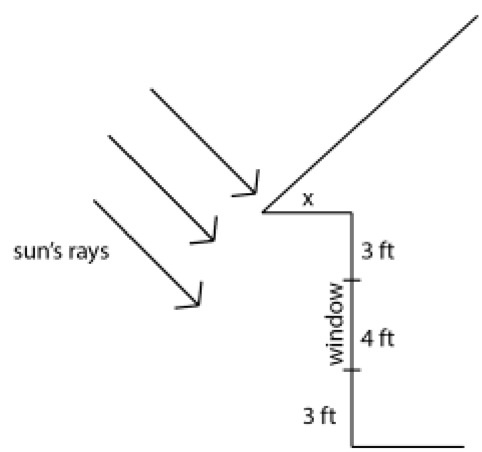
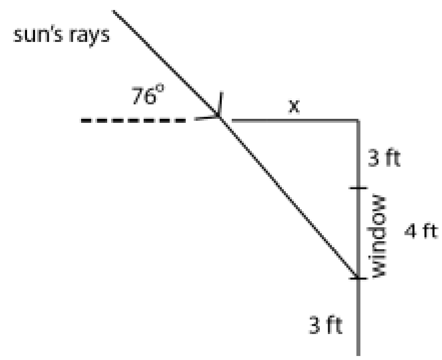



Figure 13: Demonstrates how an overhang can block the sun's rays

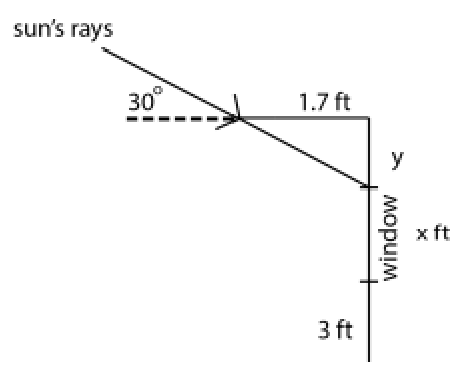
Let's return to right triangle trigonometry to solve the following problems:

*Problem 1*: Given the window placement for a house in Vilas, NC shown below, determine what the shortest overhang length that would guarantee no direct sun rays enter the window at noon on June 21st.




*Solution to Problem 1*: Because of the properties of vertical angles, the angle opposite the window is 43. So we have $\tan(76) = \frac{7}{x}$, then solving for $x$ we get $x = \frac{7}{\tan(76)} \approx 1.7\ ft$

*Problem 2:* Given the same set-up in problem 1 and an overhang length of 1.7 feet. How tall should the window be in order to let the most sun in on December 21st?




Solution: In order to find the optimal height of the window, we first need to find out how far from the ceiling the sunray will be (find $y$). Once again, we'll use tangent: $\tan(30) = \frac{x}{1.7}$, thus $x = (1.7)\tan(30) \approx 1ft$. Since the height of the room is 10 feet and the sunray is 1 foot from the ceiling, then window height should be 10-1-3=6 feet.

Homework:

1. Explain why it is hotter in the summer. (ANS: The days during the summer are longer (why?), and the summer sunlight is more intense during the summer months)

2. What information does the solar window provide? (ANS: the solar window shows the area of the sky where the sun will be between 9am and 3pm for the entire year)
3. Calculate the shortest overhang length that would guarantee no direct sun rays enter at noon on June 21st for your location. (ANS: vary based on location, but method is the same as problem 1)
4. Given the same set up in #3 and the overhang length you calculated in problem #3. How tall should the window be in order to let the most sun in on December 21st? (ANS: vary based on location, but method is the same as problem 2)

**Lesson 5**: Overhang Activity

*Activity*: Overhang Design
In this activity, students will experiment with overhang length to make best use of passive solar and passive cooling.

*Materials needed:* You will need a cardboard box (perhaps one that was used for paper with dimensions 17” across, 8.5” high, and 11” deep), ruler, scissors, marker, tape and paper.

*Directions:* Cut out a 4” by 4” window in the middle of the 17” by 8.5” side and secure (with tape) sheet(s) of paper inside the bottom of the box. On the top of the box, create an overhang (or awning) that comes out 3" to 4” (see Figure 14). You can use a piece of cardboard and tape to secure the overhang or the top fold as shown in Figure 15. Start with the lamp positioned at a height of about 18" above the table top and place this as far as possible from the box, record this distance and calculate the angle that the light reaches the floor. Measure the area of sunlit floor and record this in the table. Move the box closer and calculate the various angles at the various distances, again entering the values in the Table below.

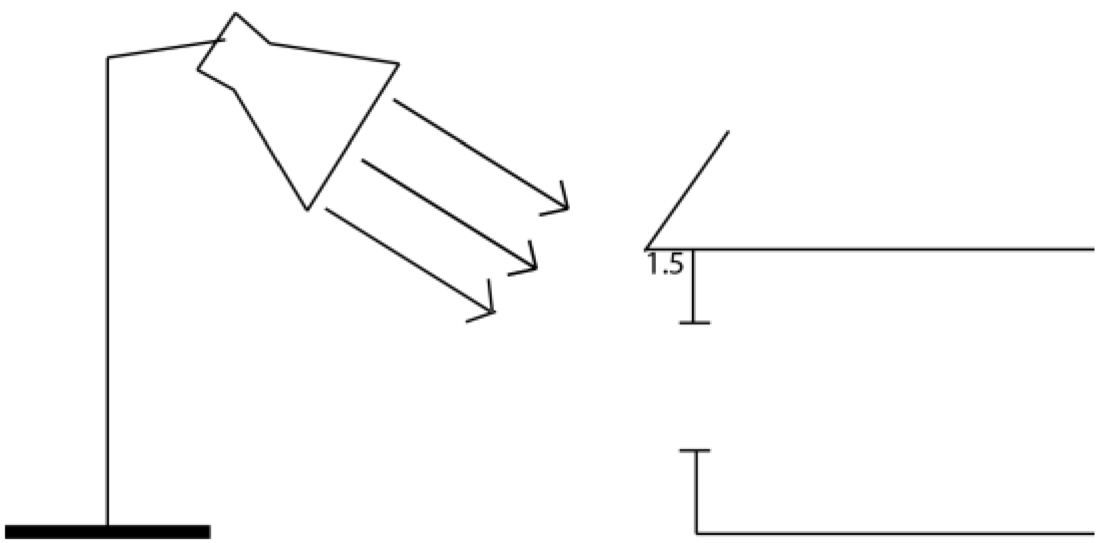



Figure 14: Heat lamp at about a 30° angle with the window

Figure 15: Actual box with overhang and lamp

Questions:

1 – Fill in the table with your data.

| Distance from box | Angle | Observed Sunlit Area (in square inches) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

2 – Compare your answers to the activity in lesson 2 (“Let the Sun Shine In”). Describe what’s happening.

3 – Compare your answers to mathematics: Using right triangle trigonometry, calculate the sunlit area when the angle is 20°, 30°, 40° and 50°. How do you answers compare?

| Angle | Floor distance when the window is in the middle of the wall | Observed Sunlit Area | Calculated Sunlit Area |
|---|---|---|---|
| 20° | | | |
| 30° | | | |
| 40° | | | |
| 50° | | | |

4 – Bringing ideas together: What is the significance of the overhang? Why is it important for passive solar?

5 – Use right triangle trigonometry (demonstrated in example 1, lesson 4) to calculate the overhang length that blocks all sun at noon on June 21st for your location.

6 – With the overhang length found in problem (5), calculate how much area of sunlit floor during the winter.

Exit Poll questions:
Overhangs are used to block out the winter sunlight. (True/False) ANS: False

Homework (Final Assessment)
In groups of 2, student will

a. Design a south facing wall given that 20% of your south-facing wall should be glass. In other words, show the size and placement of the windows. The size of the south-facing wall is 40' wide and 10' high.
b. Mathematically justify your window placement. Show that no sun comes in during summer equinox and that lots of sun comes in during the winter equinox. At noon during the summer and winter equinox, what percent of the floor is covered with sunlight?
c. Include a picture of your 30' by 40' house oriented correctly on a map (Repeat of Activity # for their location)
d. What material will you use for the floor in your house (tile(light or dark), stone, wood, laminate)? Explain your choice.
e. Create 5 questions related to passive solar that were not answered during this module. (For example – we never discuss the pros and cons of windows on the east, west and north side of the house. Do these windows matter?)

Groups will put together a 3-minute presentation about their south facing wall and window placement. Use a-e above to help design your presentation. Presentations may be done in PowerPoint or prezi.com. Physical models of design are welcomed but must be accompanied by description of the reasoning behind the design. Students will present in groups and each member's work must be obvious.

**Is it worth the cost?**
(Lesson 1, activity 1)

1. Working with a partner, make a list of the types of energy that you use throughout the day and discuss how these demands could be met by solar power. Next to each of these uses and solutions discuss whether this would be considered an active or a passive form solar energy use.

| Type of Energy | How met by solar | Active or Passive? |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

2. Payback Time: If a solar collector costs $2000 how long would the payback period be if it produces 60% of a household's hot water if the current cost of hot water is approximately $150 per month?

3. In order to do the actual analysis, one must predict future changes in the costs of conventional energy sources; what factors are likely to affect the price of hot water produced by oil or natural gas?

**Location, Location, Location**

(Lesson 1, activity 2)

In groups of 2, students should answer the following questions:

1. How do the following variables affect a site's suitability for passive solar use?
   - Latitude
   - Mean annual high and low temperatures
   - Average day length in the summer and winter

2. List at least three other variables that you would use to determine if the additional costs associated with passive solar technologies would affect your decision on whether or not to use these.
   a.
   b.
   c.
   d.

3. Visit the United States Geologic Survey's US Map (http://nationalmap.gov/ustopo/index.html) and click on the "Coverage" box.
Rank the following four locations for their likely potential for cost-effective use of passive solar space heating for a home (1 for best place, 4 for worst):

__________Key West, Florida
__________Nashville, Tennessee
__________Golden, Colorado
__________Fairbanks, Alaska

3 – Using the USGS map, mark on the map one ideal place to build a passive solar home. Using a screen clip tool, paste this map onto this document and justify your choice of location (Figure 1 is an example). Note the latitude of your site; we will use this in Lesson 4.

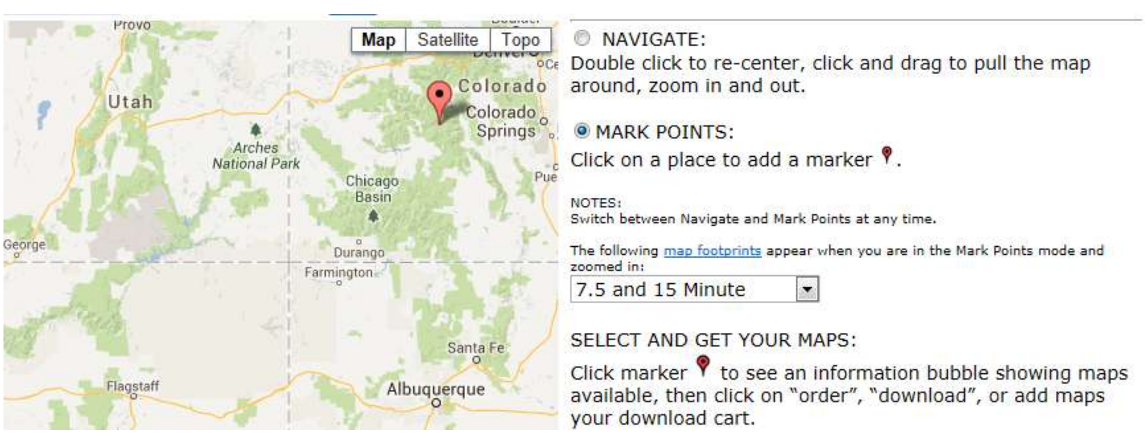



Figure (1): Marking a point on the USGS map

4. Discuss how topography (the shape of the land) affects the suitability of the site for the construction of a passive solar home.

5. Zoom in on the area selected in question 3 and select a site suitable for home construction (see Figure 2). To do this, click on the "Topo" option in the upper right hand quarter of the map and zoom in until the brown topographic contour lines appear. All USGS maps are oriented with north at the top of the map and show the elevations of points of land using these contour lines. The closer that the lines are to each other the steeper the slope, with water occurring at the bottom of a slope (symbolized by blue lines or polygons) and the high points shown by circles.


Figure (2). USGS Topographic Map showing contour lines.

**Let the Sun Shine In**
(Lesson 2 Activity)

This activity will demonstrate the relationship between the sun angle (at noon) at different times of the year with the sunlit area on the floor inside the house.

*Materials needed:* You will need a sheet of cardboard approximately 12" wide and 24" long, a ruler, scissors, marker and graph paper.

*Directions:* Lay out the cardboard as shown in Figure 1, fold it evenly in thirds, lay it flat and cut a circular hole about ½ inch in diameter in the left pane, 4 inches from the edge. Cut out a rectangle in the right hand pane, leaving about a 1 inch margin around the edge of the gap. Paste a sheet of graph paper in the center section as shown.

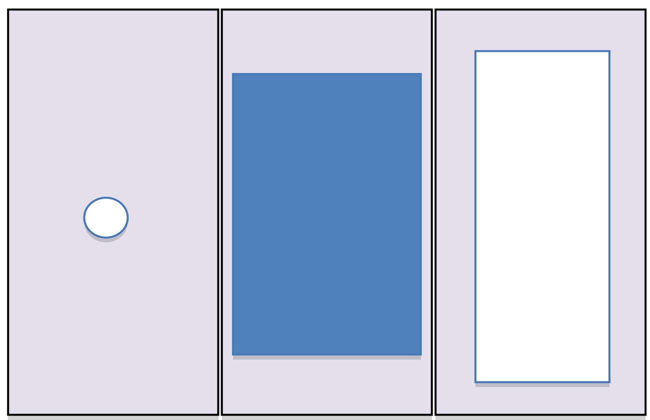


Figure 1: Construction of solar structure

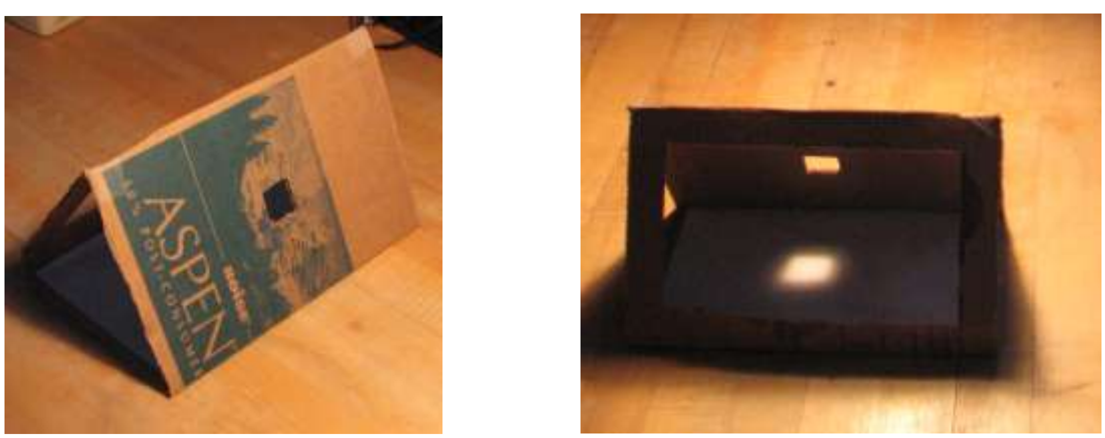



Figure 2: Folded solar structure from 2 different sides

Fold the cardboard into a right triangle and tape together (note that the side with the rectangle cut out will not reach the top), placing it on the table with the graph paper on the floor (see Figure 2). Attach a flashlight to a ring stand and place it approximately 24" from the structure (Figure 3).

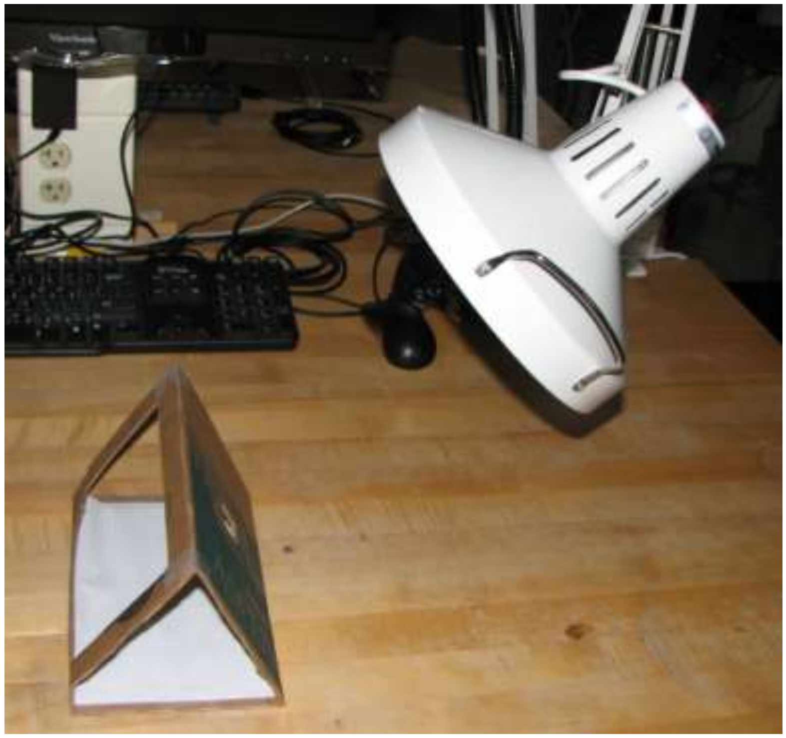


Figure 3: Folded solar structure with light

Questions:

1. Describe what happens to the sunlit area as the flashlight (sun) moves up.

2. Do you think the areas would be the same if you moved the window to the bottom of the box? Or the top of the box? Explain.

3. Quantify the floor area covered by the light shining from the flashlight. To do this, begin with the flashlight at the lowest level of ring stand and move it up until the circle of light produced on the floor touches the far wall. By counting squares on the graph paper, approximate the sun lit area. Record the height and approximate floor lit area in the first row of Table 1. Move the flashlight up the stand until no light shows on the floor and record this height (and 0 for floor lit area) in row 5 of Table 1. Fill in rows 2, 3 and 4 in even increments between the values in rows 1 and 6.

| Height of flashlight above table | Approximate floor area lit |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

Table 1. The effect of the position of the light source on the floor area illuminated

4. Does this work suggest that there is an optimal angle? In other words, is there an angle that produces the most sunlit area on the floor? If so, what is it?

5. Compare your answers to mathematics: Using right triangle trigonometry, calculate the angle of the flashlight at each of the heights and determine the expected lit area of the floor. From this, calculate the expected lit floor area. How do your answers compare?

| Calculated Angle | Predicted lit area | Actual Area |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

6. Both the area and the location of the sunlit area change as the solar angle changes. Use right angle trigonometry to determine the location of this sunlit area at the angles shown in Table 2.

| Angle of the sun | Distance of the ellipse from the far wall (in) |
|---|---|
| 15 | |
| 30 | |
| 45 | |
| 60 | |
| 75 | |

6. Bringing ideas together: Why do we care about how much sunlit area there is? We didn't consider the sunlight that hit the walls. Explain why this might matter.

## Three Little Pigs

### What material is best for an energy efficient home?

(Lesson 3 activity)

This activity investigates the different ways in which standard building materials can affect interior temperatures by comparing several identical interior spaces, represented by small, insulated cups. You will place different types of building materials inside these chests, and observe how quickly they heat up and how the temperatures changes in the cups after the light source is removed.

*Materials needed:* You will need four (or eight if increased insulation is desired), four prepared petri dish tops and four thermometers for each group. Each group will be provided with water, sand, plaster of Paris and wood.

Before doing the experiment, students should answer the following questions:

1. Predict which material will cool the air inside the box the most, before the ice chest starts warming up again from the heat gain from outside the box.

2. Will there be a time lag between the cups? In other words, which cup will get to its coldest temperature first?

Directions:

1. Fill the cups with 1" of each of the materials provided by your teacher. Cover these with the petri dish covers and place in direct sunlight or beneath a heat lamp. See Figure 1.

2. Record the temperature changes every 15 minutes for one hour.

3. Place the cups in an ice bath and record the change in temperature every 3 minutes until they have cooled to 50 degrees.

4. Plot the data for each of the four materials.

Figure 1. Cup with petri dish cover underneath a lamp

| Time (in minutes) | Temperature of Cup 1 | Temperature of Cup 2 | Temperature of Cup 3 | Temperature of Cup 4 |
|---|---|---|---|---|
| Start (0 minutes)<br>15<br>30<br>45<br>60 | | | | |
| 63<br>66<br>69<br>72<br>75 | | | | |
| 78<br>81<br>84<br>87<br>90 | | | | |
| 93<br>96<br>99<br>102<br>105 | | | | |

3. Which material heated the cup the fastest?

4. Which material maintained the temperature in the cup the longest? Working with a partner, discuss other considerations that might be important to the selection of a material for thermal mass.

*"Yesterday. All my troubles seemed so far away. Now it looks as though they're here to stay. Oh I believe in yesterday. Suddenly I'm not half the man I used to be. There's a* ***shadow hangin' over*** *me. Oh, yesterday came suddenly….." Paul McCartney*

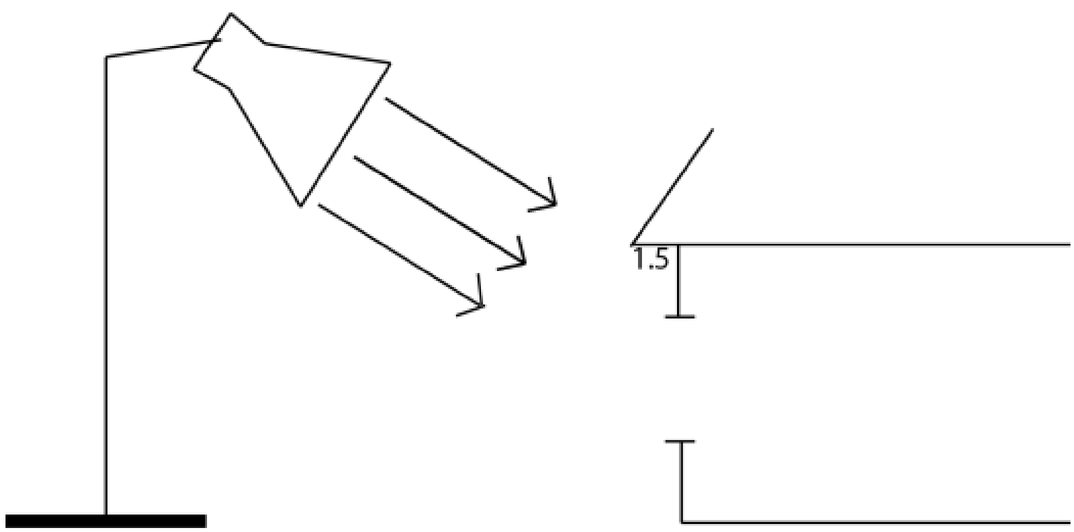
# Overhang Design

(Lesson 5 activity)

In this activity, students will experiment with overhang length to make best use of passive solar and passive cooling.

*Materials needed:* You will need a cardboard box (perhaps one that was used for paper with dimensions 17" across, 8.5" high, and 11" deep), ruler, scissors, marker and paper.

*Directions:* Cut out a 4" by 4" window in the middle of the 17" by 8.5" side and secure (with tape) sheet(s) of paper inside the bottom of the box. On the top of the box, create an overhang (or awning) that comes out 1.5". You can use a piece of cardboard and tape to secure it. Start with the lamp at approximately 20° facing the window (see Figure below). On the paper, with your marker, draw the area where direct sunlight hits, and calculate the area. Repeat this for several angles (30°, 40°, 50°, and 60°).




Questions:

1 – Fill in the table with your data.

| Angle | Observed Sunlit Area (in square inches) |
|---|---|
| 20° | |
| 30° | |
| 40° | |
| 50° | |

2 – Compare your answers to the activity in lesson 2 (“Let the Sun Shine In”). Describe what’s happening.

3 – Compare your answers to mathematics: Using right triangle trigonometry, calculate the sunlit area when the angle is 20°, 30°, 40° and 50°. How do you answers compare?

| Angle | Floor distance when the window is in the middle of the wall | Observed Sunlit Area | Calculated Sunlit Area |
|---|---|---|---|
| 20° | | | |
| 30° | | | |
| 40° | | | |
| 50° | | | |

4 – Bringing ideas together: What is the significance of the overhang? Why is it important for passive solar?

5 – Use right triangle trigonometry (demonstrated in example 1, lesson 4) to calculate the overhang length that blocks all sun at noon on June 21st for your location.

6 – With the overhang length found in problem (5), calculate how much area of sunlit floor during the winter.

## Passive Solar House Design

(Lesson 5, final Assessment)

In groups of 2, students will

a. Design a south facing wall given that 20% of your south-facing wall should be glass. In other words, show the size and placement of the windows. The size of the south-facing wall is 40' wide and 10' high.
b. Mathematically justify your window placement. Show that no sun comes in during summer equinox and that lots of sun comes in during the winter equinox. At noon during the summer and winter equinox, what percent of the floor is covered with sunlight?
c. Include a picture of your 30' by 40' house oriented correctly on a map (Repeat of Activity # for their location)
d. What material will you use for the floor in your house (tile(light or dark), stone, wood, laminate)? Explain your choice.
e. Create 5 questions related to passive solar that were not answered during this module.

Groups will put together a 3-minute presentation about their south facing wall and window placement. Use a-e above to help design your presentation. Presentations may be done in PowerPoint or prezi.com. Physical models of design are welcomed but must be accompanied by description of the reasoning behind the design. Students will present in groups and each member's work must be obvious. Students will be graded by the following rubric.

| 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|
| **Organization** | Audience cannot understand presentation because there is no sequence of information. | Audience has difficulty following presentation because student jumps around. | Student presents information in logical sequence which audience can follow. | Student presents information in logical, interesting sequence which audience can follow. |
| **Subject Knowledge** | Student does not have grasp of information; student cannot answer questions about subject. | Student is uncomfortable with information and is able to answer only rudimentary questions. | Student is at ease with expected answers to all questions, but fails to elaborate. | Student demonstrates full knowledge (more than required) by answering all class questions with explanations and elaboration. |
| **Graphics** | Student uses superfluous graphics or no graphics | Student occasionally uses graphics that rarely support text and presentation. | Student's graphics relate to text and presentation. | Student's graphics explain and reinforce screen text and presentation. |
| **Mechanics** | Student's presentation has | Presentation has three misspellings | Presentation has no more than two | Presentation has no misspellings or |

| | | | | | |
|---|---|---|---|---|---|
| | four or more spelling errors and/or grammatical errors. | and/or grammatical errors. | misspellings and/or grammatical errors. | grammatical errors. | |
| **Eye Contact** | Student reads all of report with no eye contact. | Student occasionally uses eye contact, but still reads most of report. | Student maintains eye contact most of the time but frequently returns to notes. | Student maintains eye contact with audience, seldom returning to notes. | |
| **Elocution** | Student mumbles, incorrectly pronounces terms, and speaks too quietly for students in the back of class to hear. | Student's voice is low. Student incorrectly pronounces terms. Audience members have difficulty hearing presentation. | Student's voice is clear. Student pronounces most words correctly. Most audience members can hear presentation. | Student uses a clear voice and correct, precise pronunciation of terms so that all audience members can hear presentation. | |
| | | | | **Total Points:** | |

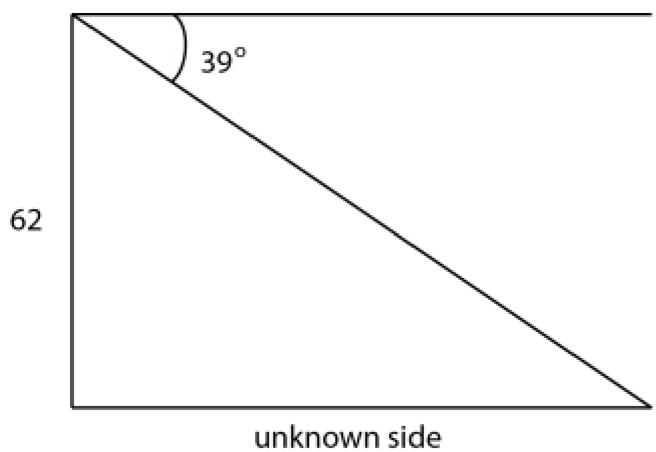
Right Triangle Trigonometry Name ___________________________

Homework for Lesson 2

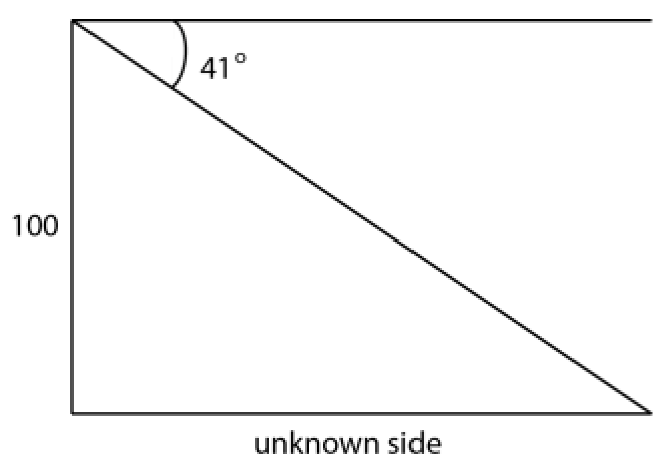
1. Sighting out his sixth floor hotel window, John finds the angle of depression to his car in the parking lot to be 39°. If his window is 62 feet above level ground, how far is his car from the hotel?
2. From the top of a lighthouse on the shore, the angle of depression to a life raft on the ocean is 41°. If the top of the lighthouse is 100 feet above sea level, how far of shore is the lifeboat?
3. From a boat on the water, the angle of elevation to the top of a lighthouse is 19.2°. If the top of the lighthouse is 150 feet above sea level, how far is the boat from the base of the lighthouse?
4. Jane sights a billboard from 200 feet away. The angle of elevation to the top of the billboard is 14°, while the angle of elevation to the bottom of the billboard is 8.5°. How tall is the billboard itself?
5. A room in a house is 12 feet deep and the length of the room is 15 feet with a 5-foot tall window that is 2 feet off the ground. If the sun's rays make an angle of 32° with the floor, what floor area is exposed to direct sunlight?
6. A room in a house is 10 feet deep and the length of the room is 15 feet with a 5-foot tall window that is 2 feet off the ground. If the sun's rays make an angle of 32° with the floor, what floor area is exposed to direct sunlight?

Right Triangle Trigonometry Name ___________________________________
Homework for Lesson 2

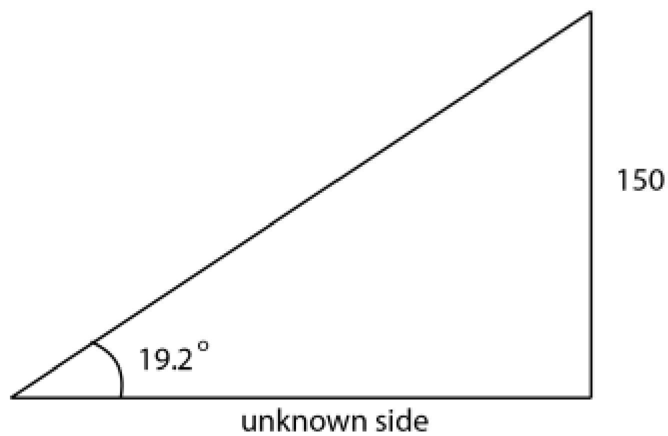
1. Sighting out his sixth floor hotel window, John finds the angle of depression to his car in the parking lot to be 39°. If his window is 62 feet above level ground, how far is his car from the hotel?



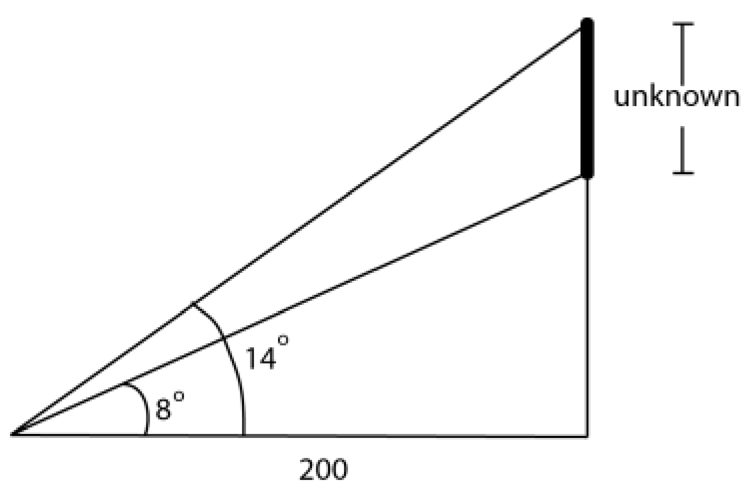

2. From the top of a lighthouse on the shore, the angle of depression to a life raft on the ocean is 41°. If the top of the lighthouse is 100 feet above sea level, how far of shore is the lifeboat?



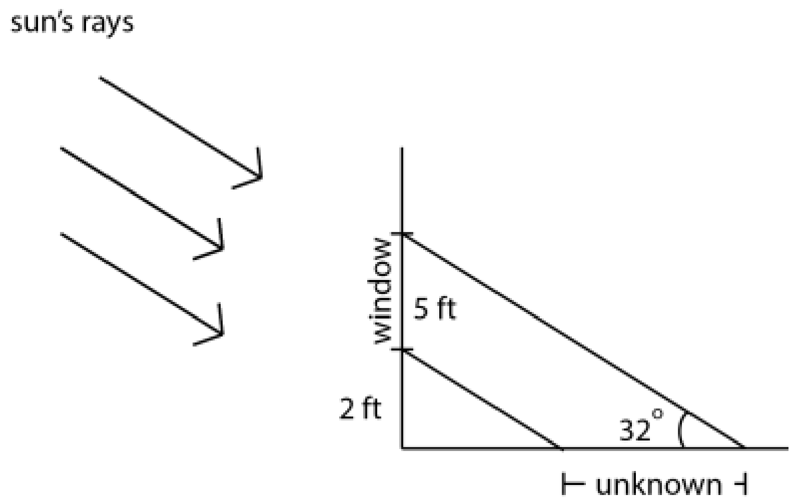
3. From a boat on the water, the angle of elevation to the top of a lighthouse is 19.2°. If the top of the lighthouse is 150 feet above sea level, how far is the boat from the base of the lighthouse?




4. Jane sights a billboard from 200 feet away. The angle of elevation to the top of the billboard is 14°, while the angle of elevation to the bottom of the billboard is 8.5°. How tall is the billboard itself?



5. A room in a house is 12 feet deep and the length of the room is 15 feet with a 5-foot tall window that is 2 feet off the ground. If the sun's rays make an angle of 32° with the floor, what floor area is exposed to direct sunlight?




6. A room in a house is 10 feet deep and the length of the room is 15 feet with a 5-foot tall and 3-foot wide window that is 2 feet off the ground. If the sun's rays make an angle of 32° with the floor, what floor area is exposed to direct sunlight? (HINT: Be sure to take the room depth into consideration)

Video Links:

Amory Lovins TED Talk: A 40-year plan for energy

http://www.ted.com/talks/amory_lovins_a_50_year_plan_for_energy.html

Saved by the Sun: a PBS documentary

http://www.pbs.org/wgbh/nova/tech/saved-by-the-sun.html

Note that the discussion of passive solar occurs near the end, beginning at minute 45, again featuring Amory Lovins